Name: …… Date: ……..

Thinking with Models _ Gradient

Investigation Task: Gradient Between Points on a Line

Assessment Criterion: B and C

Key concept :	Related Concept :	Global context:
Relationships, Change & Form	Pattern, Model & Representation	Orientation in Space and Time

Inquiry Question

How can identifying and representing patterns in coordinate geometry help us understand and communicate mathematical relationships that explain real-world changes and movements?

Objective: Students will investigate patterns in coordinate geometry by calculating and analyzing gradients between points on a line.

Context:

A group of students wanted to understand how the **gradient (slope)** changes between different points on the same straight line. They recorded several pairs of coordinates that lie on the line and calculated the gradient between consecutive points.

Tasks:

Calculate → Identify → Frame a rule → Verify → Justify & Explain

ATL Skills:

Thinking Skills: Critical thinking: Identify patterns, frame rules, justify reasoning.

Self-Management Skills: Organization: Record data systematically in tables and check calculations.

Communication Skills: Use appropriate mathematical language to express ideas

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Context: A group of students wanted to understand how the **gradient (slope)** changes between different points on the same straight line. They recorded several pairs of coordinates that lie on the line and calculated the gradient between consecutive points.

Your task is to **investigate the relationship (pattern)** between the coordinates and the gradient.

Point A (x_1, y_1)	Point B (x_2, y_2)	Calculation	Gradient (m)
(1, 3)	(2, 5)	(5 − 3) / (2 − 1) = 2 / 1	**2**
(2, 5)	(3, 7)	(7 − 5) / (3 − 2) = 2 / 1	**2**
(3, 7)	(4, 9)	(9 − 7) / (4 − 3) = 2 / 1	**2**
(4, 9)	(5, 11)	(11 − 9) / (5 − 4) = 2 / 1	**2**

1. **Calculate:**
   Complete the above table by calculating the gradient between each pair of points.

2. **Identify:**
   Identify any **patterns** you notice in the gradient values.
   What do you observe about the gradient between any two points on this line?

- The gradient (slope) is **constant** for all pairs of points.
- This means the line is **straight** with a **constant rate of change**.
- The **y-values increase by 2** each time the **x-value increases by 1**.

Pattern identified: As x increases by 1, y increases by 2. Therefore, gradient = 2.

3. **Frame a Rule:**
   Based on your observations, **frame a general rule** or **formula** for the gradient of this line.

   Since gradient = 2 and one point is (1, 3):

   Use $y=mx+c$

   $3=2(1)+c \Rightarrow c=1$

   Rule: $y=2x+1$

4. **Verify:**
   Choose two new points on the same line (for example, (6, 13) and (8, 17)) and **verify** your rule by checking if it works.

   Choose two new points on the same line: (6, 13) and (8, 17).

   Check if they follow the rule:

   - For $x = 6 \rightarrow y = 2(6) + 1 = 13$
   - For $x = 8 \rightarrow y = 2(8) + 1 = 17$

   Gradient between them:

   $$m=\frac{17-13}{8-6}=\frac{4}{2}=2$$

   Verified — the rule and gradient are consistent.

5. **Justify: Justify** your reasoning with mathematical evidence.

The gradient is **the same between any two points**, showing a **linear relationship**.

The increase in y is **proportional** to the increase in x.

Since gradient m=2, every time x increases by 1, y increases by 2.

Therefore, $y=2x+1$ correctly models the pattern.

6. **Explain** why your pattern and rule make sense using the definition of gradient.

   If the pattern continues: Next points after (5, 11):

   - When $x = 6 \rightarrow y = 13$
   - When $x = 7 \rightarrow y = 15$

Predicted points: **(6, 13)** and **(7, 15)**
Both satisfy $y=2x+1$.

Conceptual Understanding Framework

Key Concepts	Related Concepts	Global Context	Statement of Inquiry (SOI)
Relationships	Pattern	Scientific and Technical Innovation	*Discovering relationships between variables helps us understand how patterns explain real-world phenomena.*
Logic	Representation	Orientation in Space and Time	*Using logical reasoning and representations allows us to describe spatial relationships and movement accurately.*
Communication	Generalization	Identities and Relationships	*Communicating mathematical generalizations supports understanding and connection between ideas.*

ATL Category	Specific ATL Skill	How It's Applied in the Task
Thinking Skills (Critical Thinking)	Identify patterns and make generalizations	Students analyze gradients to recognize the constant rate of change and formulate a general rule.
Communication Skills	Use appropriate mathematical language to express ideas	Students communicate reasoning using precise terms like *gradient*, *linear equation*, *constant rate of change*.
Research or Self-Management Skills (Organization)	Plan and organize work effectively	Students record data in structured tables, show calculations clearly, and present findings logically.

GRASPS

Element	According to Task
G – Goal	Investigate patterns in coordinate geometry by calculating gradients between points, identify consistent relationships, formulate a general rule, and justify findings.
R – Role	You are a mathematician analyzing data to discover linear patterns and express mathematical relationships clearly.
A – Audience	Your teacher, classmates, and anyone interested in understanding the relationship between coordinates and gradients.
S – Situation	A table of coordinates is given, representing points on a line. You must explore the data, find patterns in gradients, and determine a rule that predicts y-values for any x-value.
P – Product/Performance	A completed investigation including: • Table of gradients • Identified pattern • General rule (equation of the line) • Verification using new points • Justified reasoning and clear explanation of findings.
S – Standards	Success is measured by: • Accuracy in gradient calculations • Correct identification of patterns • Clear and logical general rule • Coherent reasoning and justification • Proper use of mathematical language and representations (table, formula, verbal explanation).