

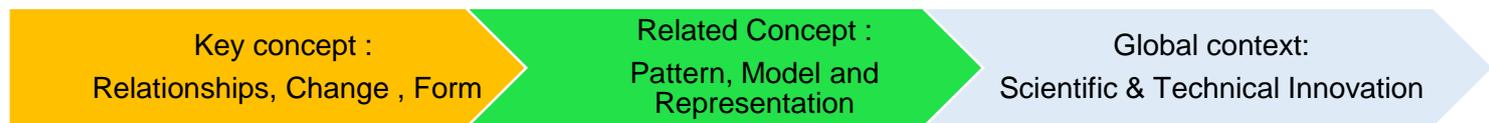
Name: .....

Date: .....

Thinking with models \_ Quadratic equations

Investigation Task: Train Motion and Braking Distance

Assessment Criterion: D and C



Inquiry Question

How mathematical models help us understand and improve transportation systems?



**Objective:** Students will be able to apply quadratic equations to model an **authentic real-life situation**. Select and use appropriate mathematical strategies to **solve and verify solutions**. Justify the **accuracy** of mathematical results and evaluate whether solutions **make sense in context**.

**Context:** In this investigation, students explore how **quadratic equations** can be used to model a **real-life train braking situation**. Students analyze the relationship between **time and distance** as a train slows down, solve a quadratic equation to find possible solutions, and interpret these solutions within a real-world context. The task emphasizes **mathematical reasoning, accuracy, and communication**, requiring students to justify both the correctness and the contextual validity of their results.



Tasks:



ATL Skills:

**Thinking Skills:** Critical Thinking: Students analyze multiple solutions and justify which one makes sense in the real-life context.

**Research Skills**  
Students interpret given information, make assumptions about braking motion, and evaluate the accuracy of the mathematical model.

**Communication Skills**  
Students explain their mathematical reasoning clearly using correct terminology, symbols, and representations.

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**Context:** A train driver applies the brakes when approaching a station. The **distance travelled after braking** depends on the train's **initial speed** and the **time taken to stop**.

The distance travelled by the train after the brakes are applied can be modeled by the quadratic equation:

$$d(t) = -2t^2 + 20t$$

Where:

- $d(t)$  is the distance travelled in metres.
- $t$  is the time in seconds after braking begins

The train must **stop exactly at the platform**, which is **48 metres** from the point where the brakes are applied.

a) Write a quadratic equation to represent when the train reaches the platform.

b) **Solve the equation** to find the possible values of time  $t$ .

c) Identify which solution is **physically meaningful** for the train. Interpret your result in the context of the train journey.

d) Explain how you know your solution is **mathematically accurate**. Verify your solution by:  
Substituting the value of  $t$  back into the original equation,

e) Explain why solution is **reasonable** and Comment on whether the model realistically represents a real train braking system

## Curriculum Framework

### Key Concepts

- 1. Relationships**  
The investigation explores the relationship between **time and braking distance**, modeled using a quadratic equation to understand how one variable affects another.
- 2. Form**  
The quadratic function shows a specific mathematical form that represents the curved pattern of a train slowing down over time.
- 3. Logic**  
Logical reasoning is used to solve the equation, verify accuracy, and decide which solution is meaningful in the real-life train context.

### Related Concepts

- 1. Modeling**  
The braking motion of a train is represented using a quadratic equation to simplify and analyze a real-life situation.
- 2. Representation**  
The investigation uses algebraic equations, numerical substitution, and possibly graphs to represent the train's motion.
- 3. Validity**  
Students evaluate whether the mathematical solutions are valid and reasonable within the physical constraints of train movement.

### Global Contexts

- 1. Scientific and Technical Innovation**  
Mathematical models are used to analyze and improve the safety and efficiency of train braking systems.
- 2. Globalization and Sustainability**  
Safe and efficient train systems support sustainable transportation and reduce environmental impact.
- 3. Orientation in Space and Time**  
The investigation focuses on motion over time and distance, helping students understand movement in physical space.

### Statements of Inquiry (SOI) Linked to Global Contexts

- 1. Scientific and Technical Innovation**  
*Mathematical relationships can be used to model motion and improve the safety of technological systems such as trains.*
- 2. Globalization and Sustainability**  
*Accurate mathematical modeling supports sustainable transportation by improving efficiency and safety.*
- 3. Orientation in Space and Time**  
*Quadratic models help us understand how objects move and stop over time and distance.*

## ATL Skills

### 1. **Critical Thinking Skills**

Students analyze multiple solutions and justify which one makes sense in the real-life context.

### 2. **Communication Skills**

Students explain their mathematical reasoning clearly using correct terminology, symbols, and representations.

### 3. **Research Skills**

Students interpret given information, make assumptions about braking motion, and evaluate the accuracy of the mathematical model.

## GRASPS

### **G – Goal**

To determine how long a train takes to stop at a platform by **solving and interpreting a quadratic equation**, and to justify the accuracy and reasonableness of the solution.

### **R – Role**

You are a **transport safety analyst** responsible for checking whether the train's braking model allows the train to stop safely at the platform.

### **A – Audience**

Train operators and safety engineers who rely on accurate mathematical models to make decisions about braking systems.

### **S – Situation**

A train applies its brakes before reaching a station, and the braking distance is modeled using a quadratic equation. The train must stop exactly at the platform to ensure passenger safety.

### **P – Product / Performance**

A **clearly structured mathematical investigation** that includes solving a quadratic equation, verifying the solution, and justifying whether the solution is realistic in the train context.

### **S – Standards for Success**

- Correct use of mathematical language and notation
- Appropriate selection and application of strategies
- Accurate verification of results
- Clear justification of contextual validity
- Logical organization and coherent communication