Name: ………………………………… Date: ………………..

Thinking with models _ Quadratic equations

Investigation Task: Train Motion and Braking Distance Assessment Criterion: D and C

Key concept : Relationships, Change , Form	Related Concept : Pattern, Model and Representation	Global context: Scientific & Technical Innovation

Inquiry Question

How mathematical models help us understand and improve transportation systems?

Objective: Students will able to apply quadratic equations to model an **authentic real-life situation.** Select and use appropriate mathematical strategies to **solve and verify solutions.** Justify the **accuracy** of mathematical results and evaluate whether solutions **make sense in context.**

Context: In this investigation, students explore how **quadratic equations** can be used to model a **real-life train braking situation**. Students analyze the relationship between **time and distance** as a train slows down, solve a quadratic equation to find possible solutions, and interpret these solutions within a real-world context. The task emphasizes **mathematical reasoning, accuracy, and communication**, requiring students to justify both the correctness and the contextual validity of their results.

Tasks:

Identify → Solve → Expain → Verify → Discuss

ATL Skills:

Thinking Skills: Critical Thinking: Students analyze multiple solutions and justify which one makes sense in the real-life context.

Communication Skills
Students explain their mathematical reasoning clearly using correct terminology, symbols, and representations.

Research Skills
Students interpret given information, make assumptions about braking motion, and evaluate the accuracy of the mathematical model.

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Thinking with models _ Quadratic equations

Investigation Task: Train Motion and Braking Distance Assessment Criterion: D and C

Context: A train driver applies the brakes when approaching a station. The **distance travelled after braking** depends on the train's **initial speed** and the **time taken to stop**.

The distance travelled by the train after the brakes are applied can be modeled by the quadratic equation: $d(t) = -2t^2 + 20t$

Where:

- d(t) is the distance travelled in metres.
- t is the time in seconds after braking begins

The train must **stop exactly at the platform**, which is **48 metres** from the point where the brakes are applied.

a) Write a quadratic equation to represent when the train reaches the platform.

$d(t) = 48$

$-2t^2 + 20t = 48$

b) **Solve the equation** to find the possible values of time t.

$-2t^2 + 20t = 48 \quad \div(-2)$

$t^2 - 10t = -24$

$t^2 - 10t + 24 = 0$

$t^2 - 6t - 4t + 24 = 0$

$t(t-6) - 4(t-6) = 0$

$(t-6)(t-4) = 0$

$t - 6 = 0$ or $t - 4 = 0$

$t = 6$ $t = 4$

c) Identify which solution is **physically meaningful** for the train. Interpret your result in the context of the train journey.

$t = 6$ $t = 4$

The train is breaking and coming to a stop

At 4 seconds, the train is still moving.

At 6 seconds, the train completes its breaking and stops.

d) Explain how you know your solution is **mathematically accurate**. Verify your solution by: Substituting the value of t back into the original equation

Substitute $t=6$ into original equation

$$d(6) = -2(6)^2 + 20(6)$$
$$= -2(36) + 120$$
$$= -72 + 120$$
$$= 48$$

The calculated distance matches the required 48m, confirming the solution is accurate.

e) Explain why solution is **reasonable** and Comment on whether the model realistically represents a real train braking system.

Time can't be negative in real life
Both solutions are positive, but only
one represents the stopping point.

Why $t=6$ make sense
* represents the end of braking
* Matches real life motion of a train slowing down.
* occurs after the train has passed intermediate points.

Why $t=4$ does not make sense
Represents a moment when the train is still moving.

$\therefore$ $t=6$ seconds is the only realistic solution.

Curriculum Framework

Key Concepts

1. **Relationships**
   The investigation explores the relationship between **time and braking distance**, modeled using a quadratic equation to understand how one variable affects another.
2. **Form**
   The quadratic function shows a specific mathematical form that represents the curved pattern of a train slowing down over time.
3. **Logic**
   Logical reasoning is used to solve the equation, verify accuracy, and decide which solution is meaningful in the real-life train context.

Related Concepts

1. **Modeling**
   The braking motion of a train is represented using a quadratic equation to simplify and analyze a real-life situation.
2. **Representation**
   The investigation uses algebraic equations, numerical substitution, and possibly graphs to represent the train's motion.
3. **Validity**
   Students evaluate whether the mathematical solutions are valid and reasonable within the physical constraints of train movement.

Global Contexts

1. **Scientific and Technical Innovation**
   Mathematical models are used to analyze and improve the safety and efficiency of train braking systems.
2. **Globalization and Sustainability**
   Safe and efficient train systems support sustainable transportation and reduce environmental impact.
3. **Orientation in Space and Time**
   The investigation focuses on motion over time and distance, helping students understand movement in physical space.

Statements of Inquiry (SOI) Linked to Global Contexts

1. **Scientific and Technical Innovation**
   Mathematical relationships can be used to model motion and improve the safety of technological systems such as trains.
2. **Globalization and Sustainability**
   Accurate mathematical modeling supports sustainable transportation by improving efficiency and safety.
3. **Orientation in Space and Time**
   Quadratic models help us understand how objects move and stop over time and distance.

ATL Skills

1. **Critical Thinking Skills**
   Students analyze multiple solutions and justify which one makes sense in the real-life context.
2. **Communication Skills**
   Students explain their mathematical reasoning clearly using correct terminology, symbols, and representations.
3. **Research Skills**
   Students interpret given information, make assumptions about braking motion, and evaluate the accuracy of the mathematical model.

GRASPS

G – Goal

To determine how long a train takes to stop at a platform by **solving and interpreting a quadratic equation**, and to justify the accuracy and reasonableness of the solution.

R – Role

You are a **transport safety analyst** responsible for checking whether the train's braking model allows the train to stop safely at the platform.

A – Audience

Train operators and safety engineers who rely on accurate mathematical models to make decisions about braking systems.

S – Situation

A train applies its brakes before reaching a station, and the braking distance is modeled using a quadratic equation. The train must stop exactly at the platform to ensure passenger safety.

P – Product / Performance

A **clearly structured mathematical investigation** that includes solving a quadratic equation, verifying the solution, and justifying whether the solution is realistic in the train context.

S – Standards for Success

- Correct use of mathematical language and notation
- Appropriate selection and application of strategies
- Accurate verification of results
- Clear justification of contextual validity
- Logical organization and coherent communication