

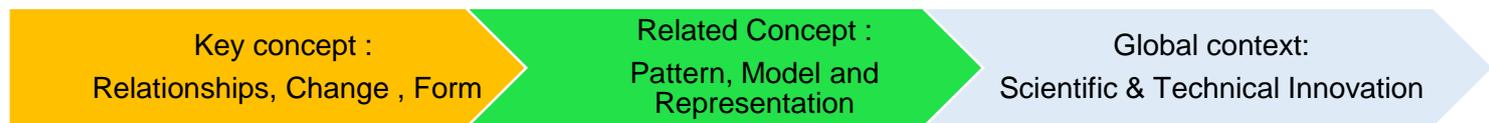
Name:

Date:

Thinking with models _ Quadratic equations

Investigation Task: Water Droplets

Assessment Criterion: B and C



Inquiry Question

How can factorization of quadratic equations help us model and predict the motion of falling water droplets?



Objective: Students will be able to apply, discover and communicate patterns in quadratic relationships by applying factorization techniques to a real-life context. Students will develop the ability to model situations mathematically, verify solutions, and clearly communicate their reasoning using multiple representations.

Context: In this investigation, students explore how quadratic equations can be used to model the motion of water droplets falling from a leaf. Students frame tables of values, identify patterns, factorize quadratic expressions, solve equations, verify solutions, and justify general rules using mathematical reasoning and appropriate representations.



Tasks:



ATL Skills:

Thinking Skills: Critical thinking: Analyzing quadratic patterns and selecting factorization as an appropriate problem-solving strategy

Organization skills
Structuring solutions logically from representation to verification and justification.



Communication Skills:
Explaining mathematical reasoning using correct notation, tables, and written justification.

Name:

Date:

Thinking with models _ Quadratic equations

Investigation Task: Water Droplets

Assessment Criterion: B and C

Context:

A scientist observes water droplets falling from a leaf. The height of a water droplet above the ground (in meters) at different times (in seconds) follows a quadratic pattern.

The height of the droplet is modeled by different quadratic expressions that can be solved using factorization to determine when the droplet reaches the ground.

The height of a water droplet is given by the quadratic expression: $h(t) = (t - 1)(t - 4)$



- a) **Frame a table** of values for $t=0, 1, 2, 3, 4, 5$. Record the corresponding height $h(t)$.

$$h(t) = (t-1)(t-4)$$

t	$h(t)$	Height
0	$(-1)(-4)$	4
1	$(0)(-3)$	0
2	$(1)(-2)$	-2
3	$(2)(-1)$	-2
4	$(3)(0)$	0
5	$(4)(1)$	4

- b) **Identify** the values of t for which the height of the droplet is zero. **State** what these values represent in the context of the water droplet

From the equation

$$(t-1)(t-4) = 0$$

$$t = 1 \text{ or } 4$$

These values represent the times when the water droplet is at ground level.

- c) **Find a rule** that explains how factorization helps determine when the droplet reaches the ground.

if a quadratic equation is written
in factored form

$$(a)(b) = 0$$

then $a = 0$ or $b = 0$

- d) **Explain** how the factors of the quadratic expression relate to the motion of the droplet.

Each factor represents a time when
the height of the droplet is zero.
Factorization directly reveals when
the droplet touches the ground.

Another water droplet is modeled by the equation: $h(t) = t^2 - 7t + 10$

- a) **Solve** the equation using **factorization**.

$$h(t) = 0$$

$$h(t) = t^2 - 7t + 10 = 0$$

$$t^2 - 7t + 10 = 0$$

$$t^2 - 2t - 5t + 10 = 0$$

$$t(t-2) - 5(t-2) = 0$$

$$(t-5)(t-2) = 0$$

$$t-5=0 \text{ or } t-2=0$$

b) **Find** the time(s) when the droplet is at ground level.

$$t = 5 \quad \text{or} \quad t = 2$$

c) **Verify** your solutions by substituting the values of t back into the original equation. Show all working clearly.

Verify $t = 2$

$$\begin{aligned} h(2) &= 2^2 - 7(2) + 10 \\ &= 4 - 14 + 10 \\ &= 0 \end{aligned}$$

Verify $t = 5$

$$\begin{aligned} h(5) &= 5^2 - 7(5) + 10 \\ &= 25 - 35 + 10 \\ &= 0 \end{aligned}$$

Both values satisfy the equation
so the solutions are correct.

d) **Justify** why factorization is an effective method for solving quadratic equations in this investigation.

Factorization is an effective method
because:

- * it clearly shows the zeroes of quadratic equation.
- * the factors directly represent real life events (when the droplet reaches the ground).

Curriculum overview

Key Concepts

- 1. Relationships**
Understanding how time and height are connected through a quadratic relationship in the motion of water droplets.
- 2. Representation**
Using tables, algebraic expressions, and equations to represent the motion of water droplets mathematically.
- 3. Logic**
applying logical reasoning to factorize quadratic equations, solve them, and justify the solutions.

Related Concepts

- 1. Patterns**
identifying repeating and predictable patterns in quadratic equations and tables of values.
- 2. Justification**
Explaining and proving why factorization leads to correct solutions in the water droplet model.
- 3. Modeling**
Using quadratic equations to model a real-life situation involving falling water droplets.

Global Contexts

- 1. Scientific and Technical Innovation**
Understanding how mathematical models explain natural phenomena such as falling water droplets.
- 2. Globalization and Sustainability**
using mathematical modeling to support water conservation and efficient irrigation systems.
- 3. Personal and Cultural Expression**
Communicating mathematical thinking clearly through different representations.

Statements of Inquiry (SOI) Linked with Global Contexts

- 1. Scientific and Technical Innovation**
Quadratic relationships can be modeled and analyzed to explain the motion of water droplets in natural systems.
- 2. Globalization and Sustainability**
Understanding mathematical patterns helps improve sustainable water management practices.
- 3. Personal and Cultural Expression**
Clear mathematical communication allows patterns and relationships to be shared and understood across cultures.

ATL Skills

- 1. Critical-thinking skills**
Analyzing quadratic patterns and selecting factorization as an appropriate problem-solving strategy.
- 2. Communication skills**
Explaining mathematical reasoning using correct notation, tables, and written justification.
- 3. Organization skills**
Structuring solutions logically from representation to verification and justification.

GRASPS – Quadratic Investigation (Water Droplets)

G – Goal

To investigate how **quadratic equations** can be used to **model and predict the motion of water droplets**, and to solve these equations using **factorization** to determine when the droplet reaches the ground.

R – Role

You are a **mathematical modeler/scientist** analyzing the motion of falling water droplets to understand their behavior using quadratic relationships.

A – Audience

Your audience is a **science research team** (or classmates and teacher) who need a clear mathematical explanation of the water droplet's motion.

S – Situation

Water droplets falling from a leaf follow a curved (parabolic) path. You are asked to use **tables, quadratic equations, and factorization** to explain and predict when the droplet reaches the ground.

P – Product / Performance

A **well-organized mathematical investigation** that includes:

- A table of values
- Factorization of quadratic equations
- Solutions and verification
- Clear justification and explanation using correct mathematical language

S – Success Criteria

Your work will be successful if you:

- Correctly solve quadratic equations using factorization
- Identify and explain patterns as general rules
- Verify and justify solutions
- Use clear mathematical language and representations
- Present information in a logical and organized structure