

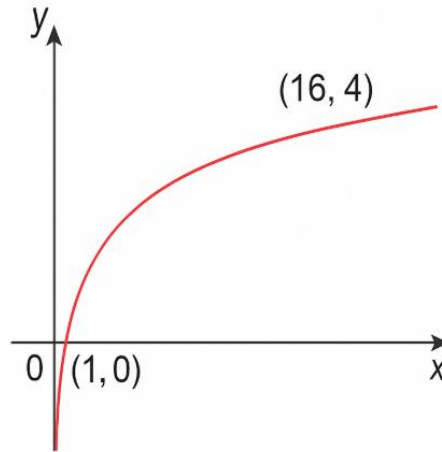
Name:

Date:

Logarithmic Function _ Exam Style Questions

1. [Maximum mark: 6]

The sketch shows the graph of $y = \log_b x$. Find the value of b .



Solution:

$$\begin{aligned} y &= \log_b x \\ x \text{ intercept is } 1 \\ \text{use } (16, 4) \\ 4 &= \log_b 16 \\ b^4 &= 16 \\ b &= \sqrt[4]{16} \\ \boxed{b = 2} \end{aligned}$$

2. [Maximum mark: 6]

Given that $g(x) = 5 - 3 \ln x$, find $g^{-1}(x)$ and state its domain.

Solution:

$$f(x) = e^{2x-3} + 5 \Rightarrow y = e^{2x-3} + 5$$

$$g(x) = 5 - 3 \ln x$$

$$y = 5 - 3 \ln x$$

To find $g^{-1}(x)$

$$x = 5 - 3 \ln y$$

$$3 \ln y = 5 - x$$

$$\ln y = \frac{5-x}{3}$$

$$y = e^{\frac{5-x}{3}}$$

$$\therefore g^{-1}(x) = e^{\frac{5-x}{3}}$$

Domain of $g^{-1}(x)$ = range of $g(x)$
Since $\ln x$ takes all real values for $x > 0$, Domain of $g^{-1}(x)$ is $(-\infty, \infty)$

3. [Maximum mark: 6]

Given that $f(x) = e^{2x-3} + 5$ find $f^{-1}(x)$ and state its domain.

Solution:

$$f(x) = e^{2x-3} + 5 \Rightarrow y = e^{2x-3} + 5$$

To find $f^{-1}(x)$, Swap x and y

$$x = e^{2y-3} + 5$$

$$x - 5 = e^{2y-3}$$

$$\ln(x-5) = 2y-3$$

$$2y - 3 = \ln(x-5)$$

$$2y = \ln(x-5) + 3$$

$$y = \frac{\ln(x-5) + 3}{2}$$

$$f^{-1}(x) = \frac{\ln(x-5) + 3}{2}$$

Domain of $f^{-1}(x)$ is $(5, \infty)$.