Name: …… Date: ……..

Composite Function_ Exam Style Questions

1. [Maximum Marks: 5] NO_GDC

Consider the functions $f(x) = 2x - 4$ *and* $g(x) = x^2 + p^2$, where p is a real constant.

a) Write down an expression for $(gof)(x)$ [2]

b) Given that $(gof)(3) = 40$, find the possible values of p. [3]

a) $g(x) = x^2 + p^2$

$(gof)(x) = g[f(x)]$ M1

$= g[2x-4]$

$= (2x-4)^2 + p^2$

$(gof)(x) = 4x^2 - 16x + 16 + p^2$ A1

b) Given that

$(gof)(3) = 40$

$(gof)(3) = 4(3)^2 - 16(3) + 16 + p^2 = 40$ M1

$36 - 48 + 16 + p^2 = 40$

$52 - 48 + p^2 = 40$ A1

$4 + p^2 = 40$

$p^2 = 40 - 4$

$p^2 = 36$

$p = \pm 6$ A1

2. [Maximum Marks: 6] GDC

Functions f and g are defined by

$$f(x) = \sqrt{x+2} \; and \; g(x) = \frac{3x-4}{x-2} \quad (x \neq 2)$$

a) Find the range of g(x) [2]

b) Solve the equations g(x) = 1 [2]

c) Find the domain and range of $(fog)(x)$ [2]

a) Graph of $g(x) = \frac{3x-4}{x-2}$

Horizontal asymptote is $y = 2$ A1

so range is $y \neq 2$ A1

b) $g(x) = 1$

$\frac{3x-4}{x-2} = 1$ M1

$3x - 4 = x - 2$

$3x - x = -2 + 4$

$2x = 2$

$x = 1$ A1

c) $(fog)(x) = f[g(x)]$

$= f\left[\frac{3x-4}{x-2}\right]$

$= \sqrt{\frac{3x-4}{x-2} + 2}$

$= \sqrt{\frac{3x-4+2(x-2)}{x-2}}$

$= \sqrt{\frac{5x-8}{x-2}}$

Critical points are $x = \frac{8}{5}$ and $x = 2$

Domain where expression $x \geq 0$ A1

$x \leq \frac{8}{5}$ or $x > 2$

Since it is square root Range is $[0, \infty)$ A1