

Name:

Date:

Composite Function_ Exam Style Questions

1. [Maximum Marks: 5]

NO_GDC

Consider the functions $f(x) = 2x - 4$ and $g(x) = x^2 + p^2$, where p is a real constant.a) Write down an expression for $(g \circ f)(x)$ [2]b) Given that $(g \circ f)(3) = 40$, find the possible values of p . [3]

$$a) \quad g(x) = x^2 + p^2$$

$$\begin{aligned} (g \circ f)(x) &= g[f(x)] \\ &= g[2x - 4] \\ &= (2x - 4)^2 + p^2 \end{aligned} \quad M_1$$

$$(g \circ f)(x) = 4x^2 - 16x + 16 + p^2 \quad A_1$$

$$b) \quad \text{Given that} \quad (g \circ f)(3) = 40$$

$$(g \circ f)(3) = 4(3)^2 - 16(3) + 16 + p^2 = 40 \quad M_1$$

$$36 - 48 + 16 + p^2 = 40$$

$$52 - 48 + p^2 = 40 \quad A_1$$

$$4 + p^2 = 40$$

$$p^2 = 40 - 4$$

$$p^2 = 36$$

$$p = \pm 6 \quad A_1$$

Functions f and g are defined by

$$f(x) = \sqrt{x+2} \text{ and } g(x) = \frac{3x-4}{x-2} \quad (x \neq 2)$$

- a) Find the range of $g(x)$ [2]
- b) Solve the equations $g(x) = 1$ [2]
- c) Find the domain and range of $(f \circ g)(x)$ [2]

a) Graph of $g(x) = \frac{3x-4}{x-2}$

Horizontal asymptote is $y=2$ A1

so range is $y \neq 2$ A1

b) $g(x) = 1$

$$\frac{3x-4}{x-2} = 1 \quad M_1$$

$$3x-4 = x-2$$

$$3x-x = -2+4$$

$$2x = 2$$

$$x = 1 \quad A1$$

$$\begin{aligned}
 c) \quad (f \circ g)(x) &= f[g(x)] \\
 &= f\left[\frac{3x-4}{x-2}\right] \\
 &= \sqrt{\frac{3x-4}{x-2} + 2} \\
 &= \sqrt{\frac{3x-4+2(x-2)}{x-2}} \\
 &= \sqrt{\frac{5x-8}{x-2}}
 \end{aligned}$$

critical points are $x = \frac{8}{5}$ and $x=2$
 domain where expression $x \geq 0$

$x \leq \frac{8}{5}$ or $x > 2$ A1
 Since it is square root Range is $[0, \infty)$ A1